

Math 002B Assignment 1.5

1. Find the general solution to the equation, then describe the solutions of the following system in parametric vector form. Also, give a geometric description of the solution set

$$x_1 + 3x_2 + x_3 = 1$$

$$-4x_1 - 9x_2 + 2x_3 = -1$$

$$3x_2 - 6x_3 = -3$$

2. Find the parametric equation of the line through **a** parallel to **b**.

$$\mathbf{a} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$$

3. Find a parametric equation of the line M through **p** and **q**.

$$\mathbf{p} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \mathbf{q} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

4. Mark each statement True or False. Justify each answer.
- a. A homogeneous equation is always consistent.
 - b. The equation $A\mathbf{x}=\mathbf{0}$ gives an explicit description of its solution set.
 - c. The homogeneous equation $A\mathbf{x}=\mathbf{0}$ has the trivial solution if and only if the equation has at least one free variable.
 - d. The equation $\mathbf{x}=\mathbf{p}+t\mathbf{v}$ describes a line through \mathbf{v} parallel to \mathbf{p} .
 - e. The solution set of $A\mathbf{x}=\mathbf{b}$ is the set of all vectors of the form $\mathbf{w}=\mathbf{p}+\mathbf{v}_h$, where \mathbf{v}_h is any solution of the equation $A\mathbf{x}=\mathbf{0}$.
4. Construct a 3×3 nonzero matrix A such that the vector $\mathbf{b}=(1,-1,1)$ is a solution to $A\mathbf{x}=\mathbf{0}$.
5. Let A be an $m \times n$ matrix, and let \mathbf{u} and \mathbf{v} be vectors in \mathbf{R}^n .
Prove that $A(\mathbf{u}+\mathbf{v})=\mathbf{0}$, and $A(c\mathbf{u}+d\mathbf{v})=\mathbf{0}$ for each pair of scalars c and d .